Fundamentals of Acoustics

Introductory Course on Multiphysics Modelling

TOMASZ G. ZIELIŃSKI

multiphysics.ippt.pan.pl

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1 Introduction

1.1 Sound waves

Sound waves propagate due to the **compressibility** of a medium ($\nabla \cdot \boldsymbol{u} \neq 0$). Depending on frequency one can distinguish (see Figure 1):

- **infrasound waves** – below 20 Hz,

- **acoustic waves** – from 20 Hz to 20 kHz,
- **ultrasound waves** – above 20 kHz.

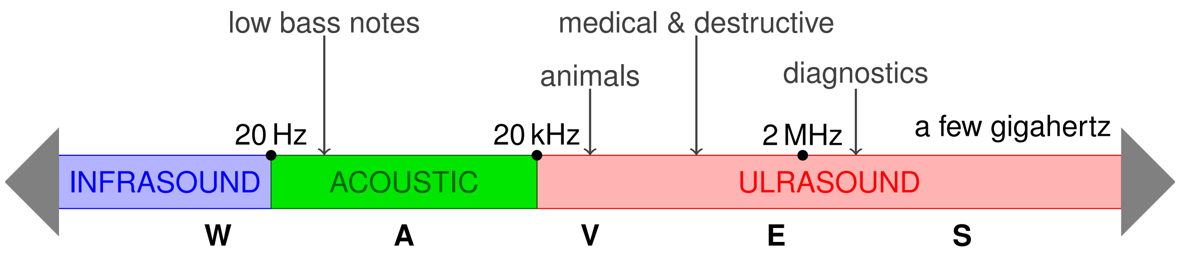

FIGURE 1: Approximate frequency ranges for sound waves and some possible application fields (*Wikipedia*).

Acoustics deals with vibrations and waves in compressible continua in the **audible frequency range**, that is, from 20 Hz (or 16 Hz) to 20 kHz (or 22 kHz).

Types of waves in compressible continua:

- an **inviscid compressible fluid** – (only) longitudinal waves,
- an infinite **isotropic solid** – longitudinal and shear waves,
- an **anisotropic solid** – wave propagation is more complex.

1.2 Acoustic variables

Particle of the fluid

It is a volume element large enough to contain millions of molecules so that the fluid may be thought of as a continuous medium, yet small enough that all acoustic variables may be considered (nearly) constant throughout the volume element.

Acoustic variables are time-varying fields defined at any point $\boldsymbol{x}$ in the fluid (i.e., for any fluid particle):

- the **particle velocity**: $\boldsymbol{u} = \dfrac{\partial \boldsymbol{\xi}}{\partial t}$,
  where $\boldsymbol{\xi} = \boldsymbol{\xi}(\boldsymbol{x}, t)$ is the particle displacement from the equilibrium position (at any point),
- the **density fluctuations**: $\tilde{\varrho} = \varrho - \varrho_0$,
  where $\varrho = \varrho(\boldsymbol{x}, t)$ is the **instantaneous density** (at any point) and ϱ_0 is the equilibrium density of the fluid,
- the **condensation**: $\tilde{s} = \dfrac{\tilde{\varrho}}{\varrho_0} = \dfrac{\varrho - \varrho_0}{\varrho_0}$,
- the **acoustic pressure**: $\tilde{p} = p - p_0$,
  where $p = p(\boldsymbol{x}, t)$ is the **instantaneous pressure** (at any point) and p_0 is the **constant equilibrium pressure** in the fluid.

Usually the gravitational forces are neglected, so that ϱ_0 and p_0 have uniform values throughout the (homogeneous) fluid.

2 Acoustic wave equation

2.1 Assumptions

General assumptions:

- **Gravitational forces can be neglected** so that the equilibrium (undisturbed state) pressure and density take on uniform values, p_0 and ϱ_0, throughout the fluid.
- **Dissipative effects**, that is viscosity and heat conduction, **are neglected**.
- The medium (fluid) is **homogeneous, isotropic, and perfectly elastic**.
- **Small-amplitudes assumption**: particle velocity is small.

Small-amplitudes assumption

Particle velocity is small, and there are only very small perturbations (fluctuations) to the equilibrium pressure and density:

$$\boldsymbol{u} - \text{small}\,, \qquad p = p_0 + \tilde{p} \quad (\tilde{p} - \text{small})\,, \qquad \varrho = \varrho_0 + \tilde{\varrho} \quad (\tilde{\varrho} - \text{small})\,. \tag{1}$$

The pressure fluctuations field $\tilde{p}$ is called the **acoustic pressure**.

These **assumptions allow for linearisation** of the following equations (which, when combined, lead to the acoustic wave equation):

The equation of state relates the internal forces to the corresponding deformations. Since the heat conduction can be neglected the *adiabatic* form of this (constitutive) relation can be assumed.

The equation of continuity relates the motion of the fluid to its compression or dilatation.

The equilibrium equation relates internal and inertial forces of the fluid according to the Newton's second law.

2.2 Equation of state

▶ PERFECT GAS

The equation of state for a perfect gas gives the thermodynamic relationship $\boxed{p = r\,\varrho\,T}$ between the total pressure p, the density ϱ, and the absolute temperature T, with r being a constant that depends on the particular fluid.

If the **thermodynamic process is restricted** the following simplifications can be achieved.

Isothermal equation of state (for constant temperature):

$$\frac{p}{p_0} = \frac{\varrho}{\varrho_0}. \tag{2}$$

Adiabatic equation of state (no exchange of thermal energy between fluid particles):

$$\frac{p}{p_0} = \left(\frac{\varrho}{\varrho_0}\right)^{\gamma}. \tag{3}$$

Here, γ denotes the ratio of specific heats ($\gamma = 1.4$ for air).

- In adiabatic process the entropy of the fluid remains constant (*isentropic* state).
- It is found experimentally that **acoustic processes are nearly adiabatic**: for the frequencies and amplitudes usually of interest in acoustics the temperature gradients and the thermal conductivity of the fluid are small enough that no significant thermal flux occurs.

▶ REAL FLUIDS

- The adiabatic equation of state for fluids other than a perfect gas is more complicated.
- It is then preferable to determine experimentally the isentropic relationship between pressure and density fluctuations: $p = p(\varrho)$.
- A Taylor's expansion can be written for this relationship:

$$p = p_0 + \left.\frac{\partial p}{\partial \varrho}\right|_{\varrho=\varrho_0} (\varrho - \varrho_0) + \frac{1}{2} \left.\frac{\partial^2 p}{\partial \varrho^2}\right|_{\varrho=\varrho_0} (\varrho - \varrho_0)^2 + \ldots \tag{4}$$

  where the partial derivatives are constants for the adiabatic compression and expansion of the fluid about its equilibrium density ϱ_0.

- If the density fluctuations are small (i.e., $\tilde{\varrho} \ll \varrho_0$) only the lowest order term needs to be retained which gives a **linear adiabatic equation of state**:

$$p - p_0 = K \frac{\varrho - \varrho_0}{\varrho_0} \quad \rightarrow \quad \boxed{\tilde{p} = K\,\tilde{s}} \tag{5}$$

  where K is the **adiabatic bulk modulus**. The essential restriction here is that the condensation must be small: $\tilde{s} \ll 1$.

2.3 Continuity equation

The **continuity equation** describes the **conservative transport of mass**. (Similar continuity equations are also derived for other quantities which are conserved, like energy, momentum, etc.)

It may be derived by considering the fluxes into an infinitesimal box of volume $\mathrm{d}V = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$, namely:

- the net flux for the x direction:
$$\varrho\,u_1 - \left(\varrho\,u_1 + \frac{\partial\varrho\,u_1}{\partial x}\,\mathrm{d}x\right)\mathrm{d}y\,\mathrm{d}z = -\frac{\partial\varrho\,u_1}{\partial x}\,\mathrm{d}V$$
- the **total influx** is the sum of the fluxes in all directions:
$$-\left(\frac{\partial\varrho\,u_1}{\partial x} + \frac{\partial\varrho\,u_2}{\partial y} + \frac{\partial\varrho\,u_3}{\partial z}\right)\mathrm{d}V = -\nabla\cdot(\varrho\,\boldsymbol{u})\;\mathrm{d}V$$

The continuity equation, see the first formula in (6), results from the fact that the total net influx must be equal to **the rate with which the mass increase** in the volume: $\frac{\partial\varrho}{\partial t}\,\mathrm{d}V$.

Linearisation of the continuity equation

$$\frac{\partial\varrho}{\partial t} + \nabla\cdot(\varrho\,\boldsymbol{u}) = 0 \;\xrightarrow[\substack{\varrho(\boldsymbol{x},t)=\varrho_0+\tilde{\varrho}(\boldsymbol{x},t)\\ \tilde{\varrho},\,\boldsymbol{u}\,-\,\text{small}}]{\text{linearization}}\; \boxed{\frac{\partial\tilde{\varrho}}{\partial t} + \varrho_0\,\nabla\cdot\boldsymbol{u} = 0} \;\xrightarrow[\tilde{\varrho}=\varrho_0\,\tilde{s}]{}\; \frac{\partial\tilde{s}}{\partial t} + \nabla\cdot\boldsymbol{u} = 0. \tag{6}$$

- The continuity equation can be integrated with respect to time
$$\int\left(\frac{\partial\tilde{s}}{\partial t} + \nabla\cdot\boldsymbol{u}\right)\mathrm{d}t = \tilde{s} + \nabla\cdot\boldsymbol{\xi} = (\text{constant} =)\,0 \quad\rightarrow\quad \tilde{s} = -\nabla\cdot\boldsymbol{\xi}\,, \tag{7}$$
where the integration constant must be zero since there is no disturbance, and $\int\nabla\cdot\boldsymbol{u}\,\mathrm{d}t = \nabla\cdot\int\boldsymbol{u}\,\mathrm{d}t = \nabla\cdot\int\frac{\partial\boldsymbol{\xi}}{\partial t}\,\mathrm{d}t = \nabla\cdot\boldsymbol{\xi}$.
- The result is combined with the adiabatic equation of state $\tilde{p} = K\,\tilde{s}$, which shows that the pressure in fluid depends on the volume dilatation $\mathrm{tr}\,\boldsymbol{\varepsilon} = \nabla\cdot\boldsymbol{\xi}$:
$$\tilde{p} = -K\,\nabla\cdot\boldsymbol{\xi} = -K\;\mathrm{tr}\,\boldsymbol{\varepsilon}\,. \tag{8}$$

2.4 Equilibrium equation

- Consider a fluid element $\mathrm{d}V$ which moves with the fluid. The mass of the element equals $\mathrm{d}m = \varrho\;\mathrm{d}V$.
- In the **absence of viscosity**, the net force experienced by the element is: $\mathrm{d}\boldsymbol{f} = -\nabla p\;\mathrm{d}V$.
- The acceleration of the fluid element (following the fluid) is the sum of the rate of change of velocity in the fixed position in space and the convective part: $\boldsymbol{a} = \frac{\partial\boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\,\boldsymbol{u}$.

- According to the Newton's second law: $\boxed{\mathrm{d}m\,\boldsymbol{a} = \mathrm{d}\boldsymbol{f}}$ which leads to the (nonlinear, inviscid) **Euler's equation**.

Momentum equation (Euler's equation)

$$\varrho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = -\nabla p \quad \xrightarrow[\substack{\varrho(\boldsymbol{x},t)=\varrho_0+\tilde{\varrho}(\boldsymbol{x},t)\\ \tilde{\varrho},\,\boldsymbol{u}\,-\,\text{small}}]{\text{linearization}} \quad \boxed{\varrho_0\,\frac{\partial \boldsymbol{u}}{\partial t} = -\nabla p} \tag{9}$$

This **linear, inviscid** momentum equation is **valid for acoustic processes of small amplitude**.

2.5 Linear wave equation

- The **linearised continuity equation**: $\dfrac{\partial \tilde{\varrho}}{\partial t} + \varrho_0\,\nabla\cdot\boldsymbol{u} = 0$ – is time-differentiated.
- The **linearised momentum equation**: $\varrho_0\,\dfrac{\partial \boldsymbol{u}}{\partial t} = -\nabla p$ – is subjected to divergence operation.
- The combination of the two transformed equations yields:

$$\frac{\partial^2 \tilde{\varrho}}{\partial t^2} - \nabla^2 p = 0 \tag{10}$$

- The **equation of state** relates the pressure to density fluctuation:

$$p = p(\tilde{\varrho}) \quad \to \quad \nabla^2 p = \frac{\partial p}{\partial \tilde{\varrho}}\,\nabla^2\tilde{\varrho} + \frac{\partial^2 p}{\partial \tilde{\varrho}^2}\,(\nabla\tilde{\varrho})^2 = \frac{\partial p}{\partial \tilde{\varrho}}\,\nabla^2\tilde{\varrho} \tag{11}$$

  - For *elastic fluids*:

$$p = p_0 + K\,\frac{\tilde{\varrho}}{\varrho_0} \quad \to \quad \frac{\partial p}{\partial \tilde{\varrho}} = \frac{K}{\varrho_0} \tag{12}$$

By using the equation of state for formula (10), the wave equation can be derived for a single unknown scalar field, for example, for the field of density fluctuations.

Wave equation for the density fluctuation

$$\boxed{\frac{\partial^2 \tilde{\varrho}}{\partial t^2} - c_0^2\,\nabla^2 \tilde{\varrho} = 0} \quad \text{where} \quad c_0 = \sqrt{\frac{\partial p}{\partial \tilde{\varrho}}} \tag{13}$$

is the **acoustic wave velocity** (or the **speed of sound**).

Notice that:

- $\varrho(\boldsymbol{x}, t) = \varrho_0 + \tilde{\varrho}(\boldsymbol{x}, t)$,
- p and $\tilde{\varrho}$ are proportional,
- $p(\boldsymbol{x}, t) = p_0 + \tilde{p}(\boldsymbol{x}, t)$,
- $\tilde{\varrho}(\boldsymbol{x}, t) = \varrho_0\,\tilde{s}(\boldsymbol{x}, t)$.

Therefore, the **wave equation** is satisfied by:

- the **instantaneous pressure**: $\frac{\partial^2 p}{\partial t^2} = c_0^2\,\nabla^2 p$
- the **acoustic pressure**: $\boxed{\frac{\partial^2 \tilde{p}}{\partial t^2} = c_0^2\,\nabla^2 \tilde{p}}$
- the **instantaneous density**: $\frac{\partial^2 \varrho}{\partial t^2} = c_0^2\,\nabla^2 \varrho$
- the **density-fluctuation**: $\frac{\partial^2 \tilde{\varrho}}{\partial t^2} = c_0^2\,\nabla^2 \tilde{\varrho}$
- the **condensation**: $\frac{\partial^2 \tilde{s}}{\partial t^2} = c_0^2\,\nabla^2 \tilde{s}$

Velocity potential

By applying the curl to the linearized momentum equation one shows that the particle velocity field is **irrotational**, $\nabla \times \boldsymbol{u} = \boldsymbol{0}$. Therefore, it can be expressed as the gradient of a scalar function $\phi(\boldsymbol{x}, t)$ known as the **velocity potential**: $\boxed{\boldsymbol{u} = \nabla\phi}$.

The **wave equation** is also satisfied by:

- the **velocity potential**: $\frac{\partial^2 \phi}{\partial t^2} = c_0^2\,\nabla^2 \phi$
- the **particle velocity**: $\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = c_0^2\,\nabla^2 \boldsymbol{u}$

2.6 The speed of sound

Inviscid isotropic elastic liquid. The pressure in an inviscid liquid depends on the volume dilatation $\operatorname{tr}\boldsymbol{\varepsilon}$:

$$p = -K\,\operatorname{tr}\boldsymbol{\varepsilon}\,, \tag{14}$$

where K is the bulk modulus. Now,

$$\frac{\partial p}{\partial t} = -K \,\mathrm{tr}\, \frac{\partial \boldsymbol{\varepsilon}}{\partial t} = -K \,\nabla \cdot \boldsymbol{u} \quad \xrightarrow[\text{Lin. Cont. Eq.}]{\nabla\cdot\boldsymbol{u} = -\frac{1}{\varrho_0}\frac{\partial \tilde{\varrho}}{\partial t}} \quad \frac{\partial p}{\partial t} = \frac{K}{\varrho_0}\frac{\partial \tilde{\varrho}}{\partial t}\,, \tag{15}$$

which means that the speed of sound $c_0 = \sqrt{\partial p / \partial \tilde{\varrho}}$ is given by the well-known formula:

$$\boxed{c_0 = \sqrt{\frac{K}{\varrho_0}}}\,. \tag{16}$$

Perfect gas. The determination of speed of sound in a perfect gas is complicated and requires the use of thermodynamic considerations. The final result is

$$c_0 = \sqrt{\gamma \frac{p_0}{\varrho_0}} = \sqrt{\gamma R T_0}\,, \tag{17}$$

where γ denotes the ratio of specific heats ($\gamma = 1.4$ for air), R is the universal gas constant, and T_0 is the (isothermal) temperature.

▶ For air at $20°$C and normal atmospheric pressure: $c_0 = 343\,\frac{\mathrm{m}}{\mathrm{s}}$.

2.7 Inhomogeneous wave equation

- The wave equation has been developed for regions of space *not* containing any sources of acoustic energy.
- However, a source must be present to generate any acoustic disturbance. If the source is *external* to the region of interest, it can be realized by time-dependent boundary conditions.
- Alternately, the acoustic equations can be modified to include *source terms*.

There are two main types of acoustic energy sources:

1. **Monopole source:** a **closed surface that changes volume** (e.g., a loudspeaker in an enclosed cabinet) – a mass is being injected into the space at a rate per unit volume $G(\boldsymbol{x}, t)$, and the *linearized continuity equation* becomes: $\frac{\partial \tilde{\varrho}}{\partial t} + \varrho_0 \,\nabla\cdot\boldsymbol{u} = G$.
2. **Dipole source:** a **body oscillating back and forth** without any change in volume (e.g., the cone of an *unbaffled* loudspeaker) – there are body forces (per unit volume) $\boldsymbol{f}(\boldsymbol{x}, t)$ present in the fluid, and the *linearized momentum equation* becomes: $\varrho_0 \frac{\partial \boldsymbol{u}}{\partial t} + \nabla p = \boldsymbol{f}$.

Taking into account *internal* sources introduces an *inhomogeneous* term into the wave equation.

Inhomogeneous wave equation

$$\frac{\partial^2 \tilde{\varrho}}{\partial t^2} - c_0^2 \nabla^2 \tilde{\varrho} = \frac{\partial G}{\partial t} - \nabla \cdot \boldsymbol{f} \quad \text{or} \quad \frac{1}{c_0^2}\frac{\partial^2 \tilde{p}}{\partial t^2} - \nabla^2 \tilde{p} = \frac{\partial G}{\partial t} - \nabla \cdot \boldsymbol{f} \quad \text{etc.} \tag{18}$$

2.8 Acoustic impedance

A general concept of impedance

Impedance can be generally described as **the ratio of a *"push"* variable** (such as voltage or pressure) **to a corresponding *"flow"* variable** (such as current or particle velocity).

- Impedance is a **frequency-domain concept**: for linear systems, if the "push" is a time-harmonic function the related "flow" must also be time harmonic, and then the time dependance cancels which makes the impedance ratio a very useful quantity – in general **frequency-dependent**.
- The "push" and "flow" variables are in general **complex**, so is the impedance.
- In certain instances, however, **it is not necessary to assume time-harmonic signals**, because the time dependence cancels regardless of the waveform. The impedance in these cases is **real and frequency-independent**. (*Example*: plane sound waves in lossless fluids.)

Definition 0 (Mechanical impedance = force/velocity)**. Mechanical impedance** of a point on a structure is **the ratio of the force** applied to the point **to the resulting velocity** at that point. **It is the inverse of mechanical admittance or mobility**, and a measure of how much a structure resists motion when subjected to a given force.

Usefulness: coupling between acoustic waves and a driving source or driven load.

Definition 0 (Acoustic impedance = pressure/velocity)**.** Two kinds of acoustic impedance are distinguished:

- **Specific acoustic impedance**: $\boxed{Z = \frac{\tilde{p}}{u}}$ $(u \equiv |\boldsymbol{u}|)$.

  It is **the ratio of the acoustic pressure** in a medium **to the associated particle speed**.

  Usefulness: transmission of acoustic waves from one medium to another.

- **Acoustic impedance** at a given surface is the ratio of the acoustic pressure averaged over that surface to the volume velocity through the surface.

  Usefulness: radiation from vibrating surfaces.

Definition 0 (Characteristic acoustic impedance)**. Characteristic impedance** of a medium: $\boxed{Z_0 = \varrho_0\, c_0}$.

For *traveling plane waves* the pressure and particle velocity are related to each other as follows:

- forward traveling waves: $\tilde{p} = Z_0\, u$,
- backward traveling waves: $\tilde{p} = -Z_0\, u$.

Typical values of characteristic impedance are given in Table 1 for air, water, and a thin aluminium rod.

TABLE 1: Density, speed of sound, and characteristic impedance for air, water, and a thin aluminium rod.

Medium	ϱ_0 $\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$	c_0 $\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$	$Z_0 = \varrho_0\, c_0$ $\left[\frac{\mathrm{Pa \cdot s}}{\mathrm{m}}\right]$
Air (at 20°C)	1.21	343	415
Distilled water	998	1482	1.48×10^6
Thin aluminium rod	2700	5050	1.36×10^7

2.9 Boundary conditions

The **acoustic wave equation** (written here for the acoustic pressure $\tilde{p}$, with monopole source $g = \frac{\partial G}{\partial t}$, and dipole source $\boldsymbol{f}$)

$$\frac{1}{c_0^2}\frac{\partial^2 \tilde{p}}{\partial t^2} - \nabla \cdot \big(\overbrace{\nabla \tilde{p} - \boldsymbol{f}}^{-\varrho_0 \frac{\partial \boldsymbol{u}}{\partial t}}\big) = g \tag{19}$$

is an example of **hyperbolic PDE**.

Boundary conditions:

1. (Dirichlet b.c.) Imposed **pressure** $\hat{\tilde{p}}$:

   $$\tilde{p} = \hat{\tilde{p}} \tag{20}$$

   For $\hat{\tilde{p}} = 0$: the *sound soft boundary*.

2. (Neumann b.c.) Imposed **normal acceleration** $\hat{a}_n$:

   $$\frac{\partial \boldsymbol{u}}{\partial t} \cdot \boldsymbol{n} = -\frac{1}{\varrho_0}\big(\nabla \tilde{p} - \boldsymbol{f}\big) \cdot \boldsymbol{n} = \hat{a}_n \tag{21}$$

   For $\hat{a}_n = 0$: the *sound hard boundary* (rigid wall).

3. (Robin b.c.) Specified **impedance** Z:

$$-\frac{1}{\varrho_0}\big(\nabla\tilde{p} - \boldsymbol{f}\big)\cdot\boldsymbol{n} + \frac{1}{Z}\frac{\partial\tilde{p}}{\partial t} = \hat{a}_n \quad \text{(usually } \hat{a}_n = 0\text{)} \tag{22}$$

For $Z = Z_0 = \varrho_0\, c_0$ (and $\hat{a}_n = 0$): the *non reflection condition* (plane waves radiates into infinity).

3 Sound levels

3.1 Sound intensity and power

The propagation of acoustic wave is accompanied by a flow of energy in the direction the wave is travelling.

Definition 0 (Sound intensity)**. Sound intensity** I in a specified direction $\boldsymbol{n}$ is defined as the time average of energy flow (i.e., power) through a unit area ΔA (perpendicular to the specified direction).

$$\text{power} = \text{force}\cdot\text{velocity} = \tilde{p}\,\Delta A\,\boldsymbol{n}\cdot\boldsymbol{u} \quad\rightarrow\quad \frac{\text{power}}{\Delta A} = \tilde{p}\,\boldsymbol{u}\cdot\boldsymbol{n}$$

$$I = \boldsymbol{I}\cdot\boldsymbol{n} = \frac{1}{t_{\text{av}}}\int_0^{t_{\text{av}}} \tilde{p}\,\boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}t \quad \text{and} \quad \boldsymbol{I} = \frac{1}{t_{\text{av}}}\int_0^{t_{\text{av}}} \tilde{p}\,\boldsymbol{u}\,\mathrm{d}t \tag{23}$$

where $\boldsymbol{u}\cdot\boldsymbol{n} = |\boldsymbol{u}| \equiv u$ if $\boldsymbol{n}$ is identical with the direction of propagation, whereas t_{av} is the averaging time. (it depends on the waveform type):

- for periodic waves t_{av} is the period,
- for transient signals t_{av} is their duration,
- for non-periodic waves $t_{\text{av}} \to \infty$.

Progressive waves of arbitrary waveform in lossless fluids:

- Forward travelling waves: $u = \frac{\tilde{p}}{Z_0}$ and then

  $$I = \frac{1}{t_{\text{av}}}\int_0^{t_{\text{av}}} \frac{\tilde{p}^2}{Z_0}\,\mathrm{d}t = \frac{\tilde{p}_{\text{rms}}^2}{Z_0} \quad \text{where} \quad \boxed{\tilde{p}_{\text{rms}} = \sqrt{\frac{1}{t_{\text{av}}}\int_0^{t_{\text{av}}} \tilde{p}^2\,\mathrm{d}t}} \tag{24}$$

  is the **root-mean-square pressure** (RMS) – for example, if $\tilde{p}$ is a sinusoidal signal of amplitude A: $\tilde{p}_{\text{rms}} = \frac{A}{\sqrt{2}}$.

- Backward travelling waves: $u = -\frac{\tilde{p}}{Z_0}$ and $I = -\frac{\tilde{p}_{\text{rms}}^2}{Z_0}$ which is negative because the energy travels in the opposite direction.

Definition 0 (Sound power)**. Sound power** W passing through a surface S is the integral of the intensity over the surface:

$$W = \int_S \boldsymbol{I} \cdot \mathrm{d}\boldsymbol{S} \tag{25}$$

3.2 Decibel scales

Sound pressures, intensities and powers are customarily described by logarithmic scales known as **sound levels**:

SPL – sound pressure level,

SIL – sound intensity level,

SWL – sound power level.

There are two reasons for doing this:

1. A wide range of sound pressures and intensities are encountered in the acoustic environment, for example:

   - audible acoustic pressure range from 10^{-5} to more than 100 Pa,
   - audible intensities range from approximately 10^{-12} to $10\,\frac{\mathrm{W}}{\mathrm{m}^2}$.

   The use of logarithmic scale compresses the range of numbers required to describe such wide ranges.

2. The relative loudness of two sounds is judged by human ear by the ratio of their intensities (which is a logarithmic behaviour).

Sound pressure level (SPL)

$$L_p = 10\log_{10}\left(\frac{\tilde{p}^2_{\text{rms}}}{\tilde{p}^2_{\text{ref}}}\right) = 20\log_{10}\left(\frac{\tilde{p}_{\text{rms}}}{\tilde{p}_{\text{ref}}}\right) \quad [\mathrm{dB}] \qquad (\text{for air: } \tilde{p}_{\text{ref}} = 20\,\mu\text{Pa}) \tag{26}$$

where $\tilde{p}_{\text{ref}}$ is a reference pressure.

- For air: $\tilde{p}_{\text{ref}} = 2 \times 10^{-5}\,\mathrm{Pa} = 20\,\mu\text{Pa}$
- For water: $\tilde{p}_{\text{ref}} = 10^{-6}\,\mathrm{Pa} = 1\,\mu\text{Pa}$

Sound intensity level (SIL)

$$L_I = 10\log_{10}\left(\frac{I}{I_{\text{ref}}}\right) \quad [\text{dB}] \qquad (\text{for air: } I_{\text{ref}} = 10^{-12}\,\tfrac{\text{W}}{\text{m}^2}) \tag{27}$$

where I_{ref} is a reference intensity. The standard reference intensity for airborne sounds is $I_{\text{ref}} = 10^{-12}\,\frac{\text{W}}{\text{m}^2}$. (This is approximately the intensity of barely-audible pure tone of 1 kHz.)

For travelling and spherical waves $I = \frac{\tilde{p}^2_{\text{rms}}}{\varrho_0\,c_0}$. Therefore, for *progressive* waves in air SPL and SIL are (in practice) numerically the same since:

$$L_I = 10\log_{10}\left(\frac{\tilde{p}^2_{\text{rms}}}{\varrho_0\,c_0\,I_{\text{ref}}}\frac{\tilde{p}^2_{\text{ref}}}{\tilde{p}^2_{\text{ref}}}\right) = \underbrace{20\log_{10}\left(\frac{\tilde{p}_{\text{rms}}}{\tilde{p}_{\text{ref}}}\right)}_{L_p} + \underbrace{10\log_{10}\left(\frac{\tilde{p}^2_{\text{ref}}}{\varrho_0\,c_0\,I_{\text{ref}}}\right)}_{-0.16\,\text{dB}} \cong L_p \tag{28}$$

Sound power level (SWL)

$$L_W = 10\log_{10}\left(\frac{W}{W_{\text{ref}}}\right) \quad [\text{dB}] \qquad (\text{for air: } W_{\text{ref}} = 10^{-12}\,\text{W}) \tag{29}$$

where W_{ref} is a reference. SWL is a measure of the total acoustic energy per unit time emitted by a source.

3.3 Sound pressure level

The root-mean-square pressure and sound pressure level for typical noise sources are given in Table 2.

TABLE 2: RMS pressure and SPL for typical noise sources.

Source (or character) of sound	$\tilde{p}_{\text{rms}}$ [Pa]	SPL [dB]
threshold of pain	100	134
hearing damage during short-term effect	20	∼120
jet engine, 100 m distant	6–200	110–140
hammer drill, 1 m distant	2	∼100
hearing damage from long-term exposure	0.6	∼85
traffic noise on major road, 10 m distant	0.2–0.6	80–90
moving automobile, 10 m distant	0.02–0.2	60–80
TV set (typical loudness), 1 m distant	0.02	∼60
normal talking, 1 m distant	0.002–0.02	40–60
very calm room	0.0002–0.0006	20–30
calm human breathing	0.00006	10
auditory threshold at 2 kHz	0.00002	0

(dB re 20µPa)

3.4 Equal-loudness contours

Figure 2 shows the so-called **equal-loudness contours** of sound pressure level. An equal-loudness contour is a measure of sound pressure (dB SPL), over the frequency spectrum, for which a listener perceives a constant loudness when presented with pure steady tones. By definition two tonal waves of different frequencies, are said to have equal-loudness level if they are perceived as equally loud by the average young person.

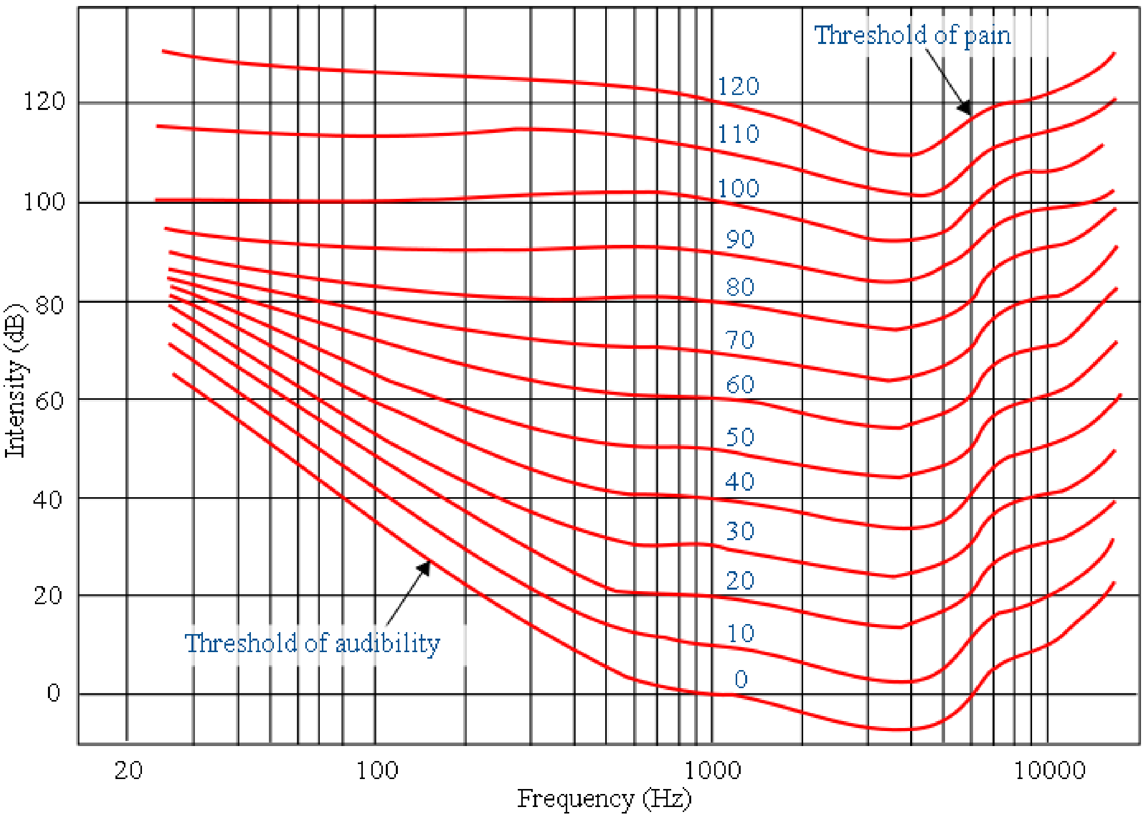

FIGURE 2: Equal-loudness contours (*Wikipedia*).

4 Absorption of sound waves

4.1 Mechanisms of the acoustic energy dissipation

- All acoustic energy is **dissipated into thermal energy**.
- Dissipation is often very slow and it can be ignored for small distances or short times.

Sources of dissipation are due to:

1. **losses at the boundaries** (relevant for porous materials, thin ducts, and small rooms);
2. **losses in the medium** (important when the volume of fluid is large). Here, the losses are associated with:

- **viscosity** – frictional losses resulting from the relative motion between adjacent portions of the medium (during its compressions and expansions);
- **heat conduction** – losses resulting from the conduction of thermal energy between higher temperature condensations and lower temperature rarefactions;
- **molecular exchanges of energy** – the conversion of kinetic energy of molecules into: stored potential energy (structural rearrangement of adjacent molecules), or internal rotational and vibrational energies (for polyatomic molecules), or energies of association and dissociation between different ionic species.

Relaxation time

Each absorption process is characterized by its **relaxation time**, that is, the amount of time for the particular process to be *nearly completed*.

4.2 A phenomenological approach to absorption

No acoustic energy loss. A consequence of ignoring any loss mechanisms is that the acoustic pressure $\tilde{p}$ and condensation $\tilde{s}$ are *in phase* as related by the **linear equation of state**:

$$\tilde{p} = \varrho_0 \, c_0^2 \, \tilde{s} \tag{30}$$

Energy loss. One way to introduce losses is to allow a *delay* between the application of a sudden pressure change $\tilde{p}_0$ and the attainment of the resulting equilibrium condensation $\tilde{s}_{\text{eq}}$, which can be yielded by a **modified equation of state** (Stokes):

$$\tilde{p} = \varrho_0 \, c_0^2 \left(1 + \tau \frac{\partial}{\partial t} \right) \tilde{s} \tag{31}$$

where τ is the **relaxation time** (see Figure 3): at $t = \tau$ the condensation reaches $1 - \frac{1}{e} = 0.632$ of its final equilibrium value $\tilde{s}_{\text{eq}} = \frac{\tilde{p}_0}{\varrho_0 \, c_0^2}$.

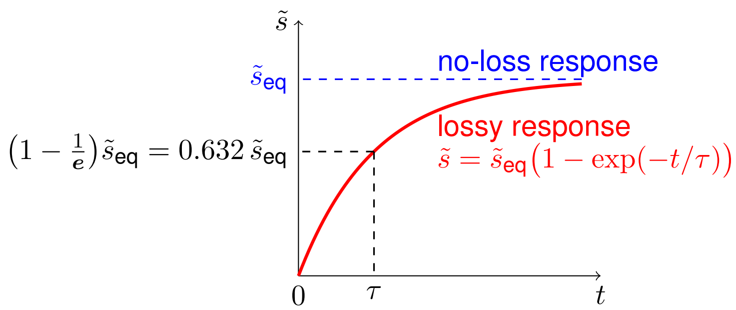

FIGURE 3: Response of a relaxing fluid to a sudden increase in pressure.

Lossy acoustic-wave equation

$$\frac{1}{c_0^2}\frac{\partial^2\tilde{p}}{\partial t^2} - \left(1 + \tau\frac{\partial}{\partial t}\right)\nabla^2\tilde{p} = 0 \tag{32}$$

Lossy Helmholtz equation

$$(\nabla^2 + k^2)\tilde{p} = 0 \quad \text{where} \quad k = \frac{\omega}{c_0}\frac{1}{\sqrt{1 + \mathrm{i}\,\omega\,\tau}} \tag{33}$$

4.3 The classical absorption coefficient

Relaxation times and **absorption coefficients** associated with:

- viscous losses (μ is the viscosity):

$$\tau_\mu = \frac{4}{3}\frac{\mu}{\varrho_0\,c_0^2} \qquad \to \qquad \alpha_\mu \approx \frac{2}{3}\frac{\omega^2}{\varrho_0\,c_0^3}\mu \tag{34}$$

- thermal conduction losses (κ is the thermal conduction):

$$\tau_\kappa = \frac{1}{\varrho_0\,c_0^2}\frac{\kappa}{C_p} \qquad \to \qquad \alpha_\kappa \approx \frac{1}{2}\frac{\omega^2}{\varrho_0\,c_0^3}\frac{\gamma - 1}{C_p}\kappa \tag{35}$$

(In gases: $\tau_\mu, \tau_\kappa \sim 10^{-10}$ s; in liquids: $\tau_\mu, \tau_\kappa \sim 10^{-12}$ s.)

Classical absorption coefficient

$$\alpha \approx \alpha_\mu + \alpha_\kappa \approx \frac{\omega^2}{2\varrho_0\,c_0^3}\left[\frac{4}{3}\mu + \frac{\gamma - 1}{C_p}\kappa\right] = \frac{\omega^2\mu}{2\varrho_0\,c_0^3}\left[\frac{4}{3} + \frac{\gamma - 1}{Pr}\right] \tag{36}$$

where $Pr = \frac{\mu C}{\kappa}$ is the **Prandtl number** which measures the importance of the effects of viscosity relative to the effects of thermal conduction.