Theoretical and Practical Introduction to *COMSOL Multiphysics*

Brief Selective Summary of the Short Course

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Website, Lecture Notes, Contact

Introductory Course on Multiphysics Modelling

`http://bluebox.ippt.pan.pl/~tzielins/index.php?im=1&id=lectures.html`

- Go to: `http://bluebox.ippt.pan.pl/~tzielins/`
- Then, choose: *Lectures*.

Suggested *Lecture Notes*:

...

6. *Introduction to Finite Element Method*
7. *Heat Transfer Problems*
8. *Galerkin Finite Element Model for Heat Transfer*

...

15. *Elementary Viscous Flow*

...

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1. Define the **problem geometry** – specify:
   - space dimension ([0D, discrete,] 1D, 2D, 3D, mixed)
   - domain or subdomains,
   - boundaries and interfaces between subdomains.

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2. Choose/derive a **mathematical model**
   - decide on **transient** (time-dependent) or **steady-state** analysis;
   - choose problem **variables**/**fields** (primary and secondary ones, eg.: concentration and flux, or temperature and heat flux vector);
   - use or derive **model equations** (usually in terms of Partial Differential Equations, e.g., the diffusion equation);
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4. **Post-process the results** of solution and **draw conclusions** from the model predictions (re-design, optimise, etc.).

Basics of multi-scale modelling

Motivation:

- **Many complex phenomena** involve processes occurring at different scales (of space and/or time), or . . .
- . . . **multiple spatial and/or temporal scales can be distinguished** to differ between the process phases or to better/easier describe the process features.
- Usually, **it is easier to deal with different scales individually**.

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Multi-scale modelling

Mathematical solution techniques of dealing with problems which have important features at multiple scales of space and/or time.

Comment: For many problems, the processes (i.e., sub-problems) at various scales can be, in practice, solved (quasi) separately, which makes such multi-scale approach very efficient.

Basics of multi-scale modelling

Multi-scale modelling

Mathematical solution techniques of dealing with problems which have important features at multiple scales of space and/or time.

Requirements:

- **Separation of scales** – allows to apply different approaches to treat problems at various scales. One can distinguish:
  different spatial scales – when there are local and global phenomena, or there co-exist processes which are: essentially microscopic (i.e., occur at the micro-scale), mesoscopic (i.e., occur at the meso-scale), and macroscopic (i.e., occur at the macro-scale), etc.;
  different temporal scales – when the involved processes are: relatively slow (static or quasi-static), dynamic, or relatively fast, etc.

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- **Representativeness** of a geometry or time-interval for the phenomenon considered at the scale related to this geometry or time-interval.

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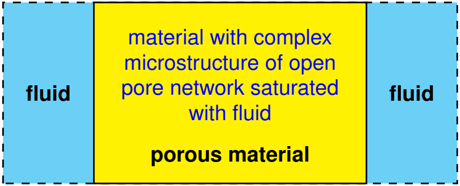
  - **different temporal scales –** when the involved processes are: relatively slow (static or quasi-static), dynamic, or relatively fast, etc.
- **Representativeness** of a geometry or time-interval for the phenomenon considered at the scale related to this geometry or time-interval.
- Well defined way of **passing of the relevant information** (effective properties, behaviour, etc.) **between the scales**.

Basics of multi-scale modelling

EXAMPLE: Transport through a porous medium

MACRO-SCALE

viscous flow through a porous material



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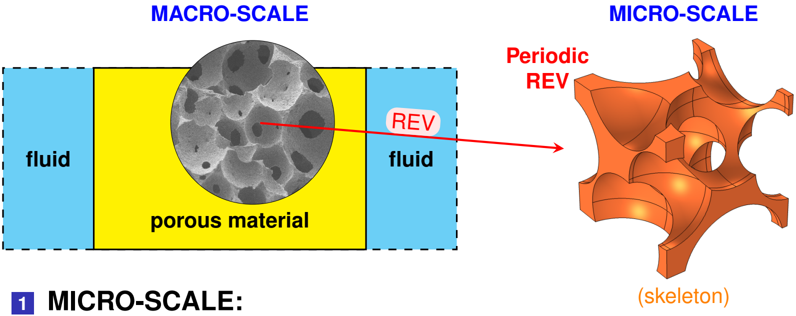

1 MICRO-SCALE:

- Selection (construction) of a (periodic) **Representative Elementary Volume (REV)** of a porous medium.

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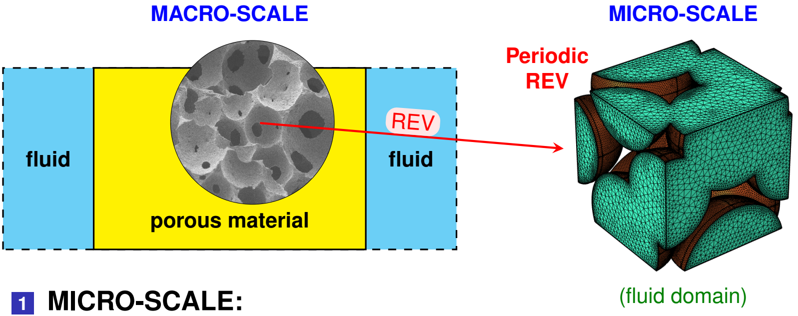

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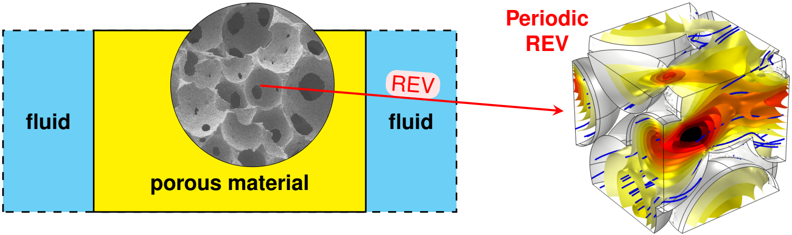

(Stokes flow)

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- **Stokes flow**, i.e., linear & steady, viscous, incompressible flow through the **periodic RVE**, driven by a uniform pressure gradient.

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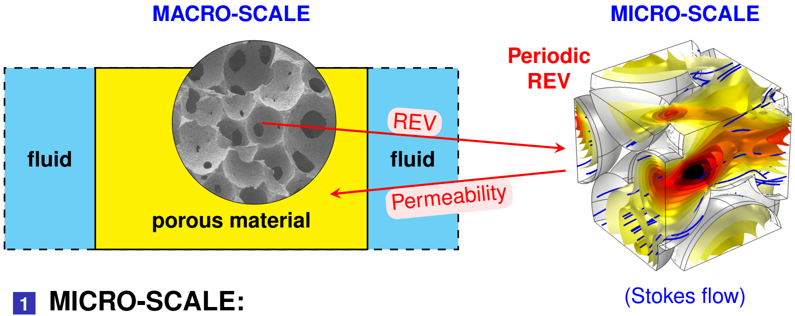

1 MICRO-SCALE:

- Selection (construction) of a (periodic) **Representative Elementary Volume (REV)** of a porous medium.
- **Stokes flow**, i.e., linear & steady, viscous, incompressible flow through the **periodic RVE**, driven by a uniform pressure gradient.
- Averaging of the computed velocity field to determine the **permeability of the porous medium**.

2 MACRO-SCALE:

- **Macroscopic flow through the porous material** characterised by its open porosity and permeability using the **Darcy's law**.

Questions & Answers

1. *What is steady-state (stationary) problem?*

   A system is in **steady state** if its recently observed behaviour will continue into the future, so that time can be eliminated from the problem description, which means that the corresponding **stationary** problem is **time-independent** (i.e, the problem variables do not depend on time). Examples: static problems, steady-state flow, time-harmonic problems.

Questions & Answers

1. *What is steady-state (stationary) problem?*
2. *What is transient (non-stationary) problem?*

   A system is in **transient state** where it substantially changes over time, which means that the problem is essentially **time-dependent** (i.e, the problem variables depend on time). The transient state is often a start-up in many steady state systems.

Questions & Answers

1. *What is steady-state (stationary) problem?*
2. *What is transient (non-stationary) problem?*
3. *What is Boundary Value Problem?*

   A **Boundary Value Problem (BVP)** is a **Partial Differential Equation (PDE)** – defined on a specified domain – together with appropriate **boundary conditions** – defined on the domain boundary. BVPs are (time-independent) mathematical models of most steady-state physical phenomena.

Questions & Answers

1. *What is steady-state (stationary) problem?*
2. *What is transient (non-stationary) problem?*
3. *What is Boundary Value Problem?*
4. *What is Initial Boundary Value Problem?*

   An **Initial Boundary Value Problem (IBVP)** is defined by a **time-dependent Partial Differential Equation (PDE)** with appropriate **initial and boundary conditions**. IBVPs are (time-dependent) mathematical models of most transient physical phenomena.

Questions & Answers

1. *What is steady-state (stationary) problem?*
2. *What is transient (non-stationary) problem?*
3. *What is Boundary Value Problem?*
4. *What is Initial Boundary Value Problem?*
5. *What are mechanisms of heat transfer?*

   Three mechanisms of heat transfer:

   I. the **conduction** – the heat transfer by diffusion,
   II. the **convection (advection)** – the heat transfer due to the bulk movement of fluid,
   III. the **radiation** – the heat transfer via electromagnetic waves.

Questions & Answers

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5. *What are mechanisms of heat transfer?*
6. *What is a usual modelling procedure using Finite Element Method?*

   Major steps in modelling using Finite Element Method:

   I. Define the **problem geometry**: decide on 2D or 3D; if possible, take advantage of symmetry; construct domain (or subdomains) with well-defined boundaries (and interfaces).
   II. Choose a **mathematical model**: decide on steady state (BVP) or transient state (IBVP); specify material(s), sources (or excitations), boundary conditions (and initial conditions in the case of IVBP).
   III. Construct (or generate) a **finite element mesh**.
   IV. **Solve** the problem numerically (choose a numerical solver; in the case of IBVP set the time scope and time step).
   V. **Post-process** and interpret the results (draw conclusions, etc.).